Superfluid T_c of Helium-3 and its Pressure Dependence

Yatendra S. Jain

Department of Physics
North-Eastern Hill University, Shillong - 793 022, India

Abstract

Superfluid T_c of liquid helium-3 and its pressure dependence are calculated by using a relation obtained from our macro-orbital microscopic theory. The results agree closely with experiments. This underlines the accuracy of our relation and its potential to provide superfluid T_c of electron fluid in widely different superconductors and renders experimental foundation to our conclusion (*cond-mat/0603784*) related to the basic factors responsible for the formation of (**q**, **-q**) bound pairs of fermions and the onset of superfluidity in a fermionic system. Since available experimental studies of superconductors pertaining to changes in lattice parameters around their superconducting T_c seem to support a link between lattice strain and the onset of superconductivity, need for similar studies is emphasized.

1. Introduction

Liquid ^3He has been a subject of extensive theoretical and experimental studies [1-5] for the last six decades for several reasons including its superfluid behavior and it appears that it will continue to fascinate the researchers for many more decades to come. However, as remarked by Georges and Laloux [6] several aspects of even normal state of the liquid at low temperatures (LT), *viz.*: (i) increase in inertial mass as revealed by the experimental values of its LT specific heat, (ii) many fold increase in its magnetic susceptibility indicating as if it is at the blink of ferromagnetic instability, and (iii) nearly temperature independent low compressibility need better understanding. Two models, *viz.*: (i) "almost ferromagnetic" [7] and "almost localized" [8,9] have been extensively tried to account for these aspects. Identifying that the two models are seemingly contradictory, Georges and Laloux [6] propose Mott-stoner liquid model. However, in view of our recently developed microscopic model of a *system of interacting fermions* (SIF) used to conclude the basic foundations of superconductivity [10] both these pictures coexist. We, therefore,

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have a detailed program to study different aspects of liquid ^3He (*including those listed above*) in the framework of our model which is based on the macro-orbital representation of a particle in a many body system. We also use this representation to conclude the long awaited microscopic theory a system of interacting bosons such as liquid ^4He [11].

It is evident that the results and inferences of our model [10] can be applied to understand the normal and superfluid behavior of liquid ^3He. We note that some of our conclusions to some extent agree with those of the well known BCS theory [12] of superconductivity. For example we find that superconductivity is a consequence of *bound pairs* of electrons moving with equal and opposite momenta ($\mathbf{q}$, -$\mathbf{q}$). But in variance with BCS theory, the binding of such pairs is basically found [10] to be a consequence of the mechanical strain in the lattice forced by the zero-point force of electrons arising from their zero point energy; the electrical polarization of the lattice emphasized by the BCS model may have its $+ve$ or $-ve$ contribution to this binding. In addition our approach reveals a single theoretical framework for the superconductivity of conventional as well as high T_c systems and finds that superconductivity can, in principle, be observed at temperatures as high as room temperature (RT). It renders mathematically simple formulations and microscopic foundations to the well known phenomenological theories (*viz.* two fluid theory of Landau [13] and $\Psi-$theory of Ginzburg [14]). It concludes that superfluid and superconducting transitions are a kind of *quantum phase transitions* which, however, occur at a non-zero T due to proximity effect of quasi-particle excitations. Guided by all these factors, we use our approach to: (i) estimate the value of superfluid T_c of liquid ^3He which has been identified to be a difficult problem [1], (ii) study its pressure dependence, and (iii) analyze their consistency with experiments. The details of other important properties of the liquid would be analyzed in our forthcoming paper(s).

Theoretical calculations, predicting possible value(s) of T_c of superfluid ^3He, based on BCS picture were reported within a year of the publication of the BCS theory. While the first few studies [15, 16] indicated that the liquid was unlikely to have a superfluid transition, Emery and Sessler [17] concluded that a second order transition may occur at a T between 50 to 100 mK. However, when the transition was really observed between 0.92 to 2.6 mK [18] (depending on the pressure on the liquid), calculations by Levin and Valls [19, 20] not only obtained a T_c close to these values but also found its pressure dependence closely matching with experiments. Almost similar estimates have been reported recently by Rasul and coworkers [21, 22]. Widely different inferences and estimated T_c from different theoretical calculations using common picture (BCS Theory) as their central idea seem to indicate the complexity of estimates and lack of reliability. On the other hand, the merit of our theory [10] lies with the fact that it does not have any scope to use different considerations to obtain different T_c which indicates its reliability. In addition the fact that our T_c for superfluid ^3He its pressure dependence agrees closely with experiments indicates its accuracy.

2 Superfluid T_c and its Pressure Dependence

Using the universal component ($H_o(N)$, Eqn. 2 of [10]) of the net hamiltonian $H(N)$ of a SIF (Eqn. 1 of [10]), such as electron fluid in a conductor or liquid ^3He, we find that
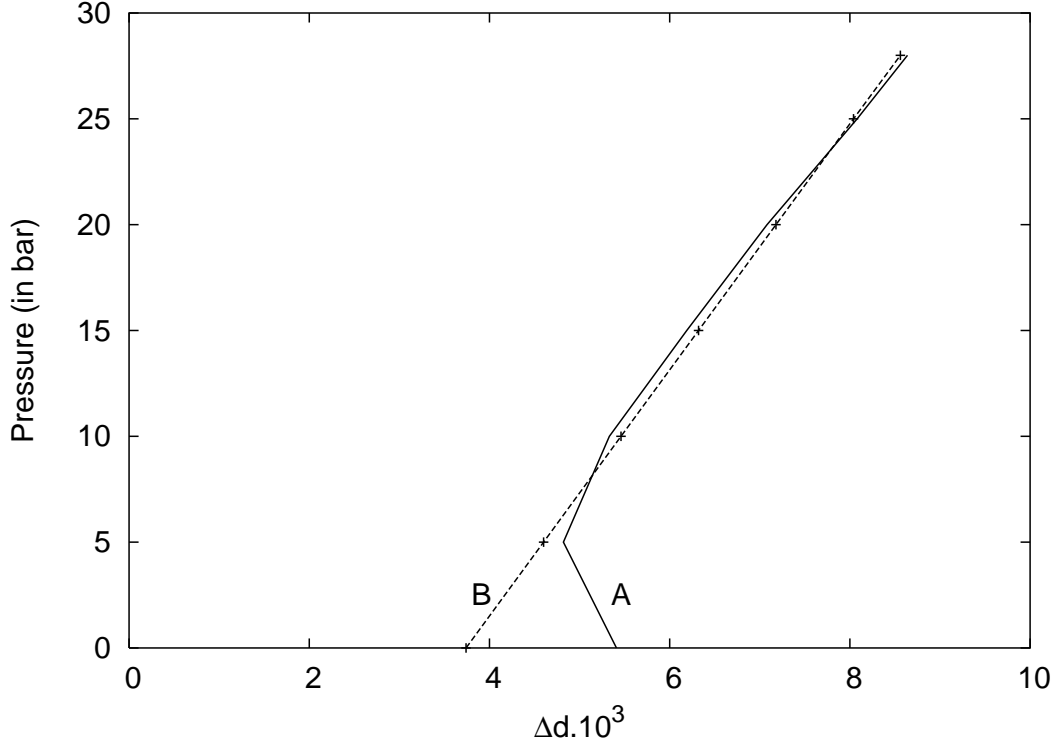
Fig. 1 : Pressure dependence of expansion in $He - He$ bond Δd (in Å). While Curve A is obtained from the molar volume data of Kollar and Vollhardt [24], Curve B represents the linear fit (Eqn. 3) after excluding point P=0 at Curve A (see text).

its particles below

$$T_c \approx \epsilon_g/k_B = \frac{h^2}{8k_Bmd^2}\frac{\Delta d}{d} \tag{1}$$

assume a state of bound pairs and the system as a whole has a transition to its superfluid state [10]. ϵ_g and m in Eqn. 1, respectively, represent the binding energy (or energy gap) and mass of a fermion with (i) $d = (V/N)^{1/3)}$ and (ii) $\Delta d = $ an increase in d forced by the zero-point force of a fermion occupying its ground state in a cavity (size $= d$) formed by neighboring fermions. It may be noted that for electrons in a conductor d in Eqn. 1 represents diameter d_c of the channels *through which conduction electrons move in the lattice* [10]. In view of the fact revealed from the experimentally observed specific heat values of liquid ^3He, a particle in an interacting environment of the liquid at a T closed to T_c starts behaving like a quasi-particle of mass m^*, we use

$$T_c = \frac{h^2}{8k_Bm^*d^2}\frac{\Delta d}{d} \tag{2}$$

to obtain T_c of liquid ^3He at different pressures. In this we define $\Delta d = d(T = 0) - d_{min}$ with $d_{min} = d$ at the point of maximum density of the liquid for a chosen pressure. As shown for the simple case of a particle trapped in 1-D box [23], we identify [10] the zero-point force of a particle occupying its ground state in the cavity of neighboring particles as the microscopic reason for the expansion of the liquid on cooling below certain $T < T_F$

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(Fermi temperature). We determine d and Δd by using molar volume of the liquid recently reported by Kollar and Vollhardt [24]. However, as indicated by Kollar and Vollhardt [24] themselves and the plot of Δd *vs.* T in Figure 1, their data for $P = 0$ seem to have large systematic errors; note that Δd at $P = 0$ falls considerably away from any logical trend in which other points can be fitted. Consequently, we discarded this point and obtained a linear fit

$$P = 5806.15\Delta d - 21.6865 \tag{3}$$

for all other points by using a standard computer software. In this context not only the remaining points seem to fall closely on the line but a linear change in Δd with increasing P is also expected because Δd is a kind of strain in $He - He$ bonds. As such we used Eqn.3 to obtain Δd values for our calculations of T_c at different pressures including $P = 0$. To obtain m^* that enters in Eqn. 2, we note that as per our theoretical formulation the quasi-particle excitations which contribute to the specific heat of the fermionic system of non-interacting particles have $4m$ mass. Obviously, when the impact of interactions is included, we have $4m^*$ as the mass of the quasi-particle which, obviously, equals the effective mass (m^*_{sp}) that we obtain from specific heat data [25]. In other words we use $m^* = m^*_{sp}/4$ in Eqn. 2 to obtain our T_c at which the superfluid phase transition is expected as per our theory [10]. The T_c values so obtained are tabulated with experimental values in Table I and both are plotted in Fig. 2 for their comparison. The fact that our m^* changes from $0.7525m(^3He)$ to $1.4233m(^3He)$ with pressure increasing from 0 to 28.0 bar indicates that inter-particle interaction dominated locally by zero-point repulsion slowly assumes attractive nature (at ≈ 10 bar pressure) with ^3He atoms having increased electric dipolemoment with increasing pressure.

Table I : Calculated and experimental T_c and related data

Pressure (in bar)	d $\mathring{A}$	Δd $\mathring{A}$	m^*_{sp} [+] $m(^3He)$	m^* $m(^3He)$	$T_c(eqn.2)$ mK	$T_c(Exp)$ [++] mK
0.0	3.94	.00374	3.010	0.7525	2.0564	0.92
5.0	3.78	.00460	3.629	0.9073	2.3667	1.60
10.0	3.69	.00546	4.183	1.0458	2.6208	1.99
15.0	3.63	.00632	4.670	1.1675	2.8598	2.21
20.0	3.58	.00718	5.084	1.2710	3.1057	2.37
25.0	3.54	.00804	5.472	1.3680	3.3399	2.47
28.0	3.52	.00856	5.693	1.4233	3.4782	2.52

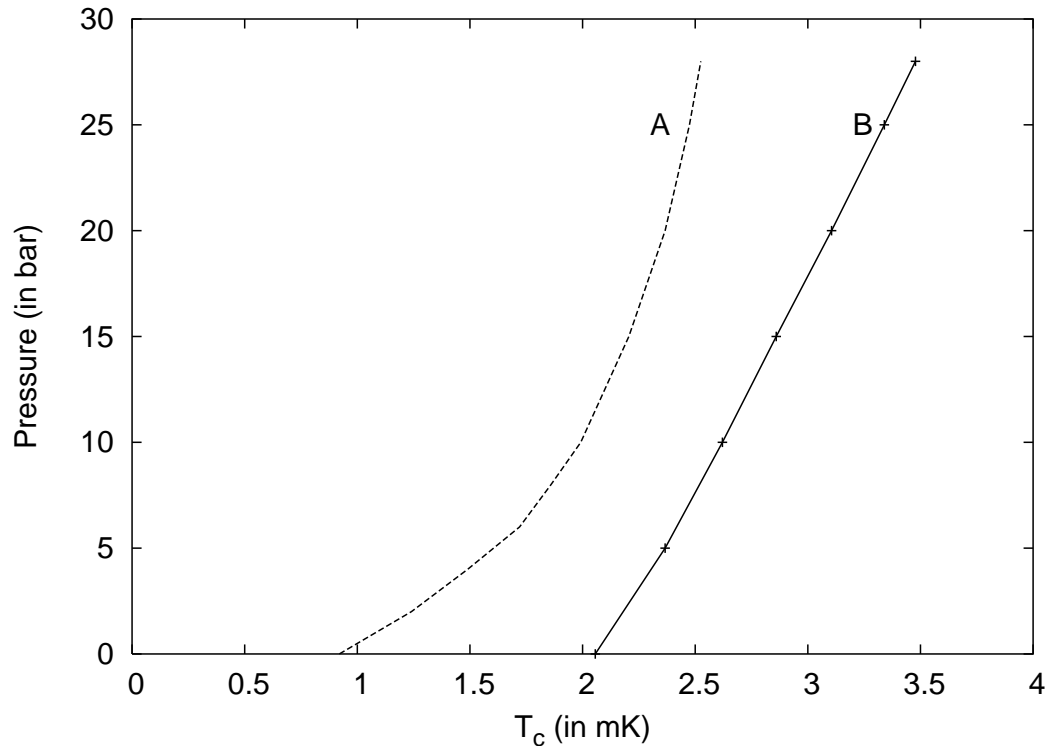[+] obtained from graphical plots of m^*_{sp} values [25], [++]Zero pressure value from [26] and others from [25].

Fig. 2 : Pressure dependence of superfluid T_c of liquid ^3He. Curve A (experimental, *cf.* Column 7, Table I) and Curve B (Eqn 2).

3. Discussion

The BCS model, basically formulated to explain superconductivity of conventional superconductors, has been used to understand superfluidity and related aspects of liquid ^3He because the electron fluid in conductors and liquid ^3He are closely identical SIF; of course suitable modifications (*e.g.*, fermions participating in Cooper pairing in the latter case have $p-$state not $s-$state) compatible with the model are adopted. This paper uses the same considerations to apply the basic foundations of superconductivity [10] revealed from our non-conventional theoretical framework which emphasizes mechanical strain (*in the crystalline lattice of a superconductor or in inter-atomic bonds in case of liquid ^3He type* SIF) as the main source of ($\mathbf{q}$, $\mathbf{-q}$) bound pair formation. As established in [23, 27], such strain is *a basic consequence of zero-point force arising from the wave particle duality* and it ought to be present whenever a particle occupies its ground state in a box or cavity of its neighboring particles or in a channel through which it is free to move. While electrons in superconductors create strain in the lattice structure of the channels through which they move [10], a He atom creates this strain in $He - He$ bonds which it makes with its neighboring atoms [11]. The experimental fact that liquid ^3He as well as liquid ^4He show $-ve$ thermal expansion at $T \approx T_o$ (*the temperature equivalent of the ground state energy of He atom in a cavity of neighboring atoms*) confirms the presence of strain in $He - He$ bonds. Evidently, our theoretical estimate of superfluid T_c of liquid ^3He and its pressure dependence (*cf.*, Table 1 and Figure 2), which have close agreement with experiments [25,26], undoubtedly prove the accuracy of Eqn. 2 and conclusions of [23 and 27]. It also demonstrates the potential of our theory [10] to predict

the superfluid T_c of a SIF which has been a difficult task in the framework of conventional BCS theory [1]. In other words Eqn. 2 can be used to estimate the superconducting T_c of widely different superconductors (including *high T_c superconductors*) if accurate values of d, Δd and m^* are known. Several experimental studies [*e.g.*, 28-33] indicate that the occurrence of lattice strain or related effects such as negative expansion of lattice, hardening of lattice, anomalous or anisotropic change in lattice parameters, *etc.* around superconducting T_c are common aspects of superconductors. This naturally supports our inference [10] regarding the relation between lattice strain and bound pair formation. However, the effect is not seen to be as clean as in liquid ^3He because an electron in a superconductor not only interacts with other electrons but also to the ions or atoms which decide their lattice arrangement through complex inter-particle interactions. Naturally, an accurate prediction of superconducting T_c from Eqn.2 depends on the accuracy of the experimentally measured $\Delta d/d$, d and m^* for a chosen superconductor. In view of Eqn.2, T_c increases with increase in $\Delta d/d$ and decrease in d and m^* and, depending on the values of these parameters, superfluid transition in a SIF can, in principle, occur at any temperature. This is corroborated by the facts that: (i) an atomic nucleus exhibits nucleon superfluidity at a T much higher than even room temperature because nucleon-nucleon d is found to be about 10^{-5} times shorter than $^3He - ^3He$ distance which implies that the typical T_c should be as high as 10^7K (the mass of a nucleon and ^3He atom having same order of magnitude), and (ii) a typical superconducting T_c falls around 10 K ($\approx 10^3$ times the superfluid T_c of liquid ^3He) because m_e (mass of electron) is about 6000 times smaller than $m(^3He)$ (inter-electron distance or the channel size being nearly equal to $^3He - ^3He$ distance).

We note that Eqn.2 has been obtained by analyzing the universal component, $H_o(N) = H(N) - V'(N)$, of the net hamiltonian $H(N)$ (*cf.*, Eqn. 1 of [10]) of a SIF with $V'(N)$ representing the sum of all non-central potentials including spin-spin interactions; as such it considers only bare fermion-fermion central forces. Evidently, our estimates of T_c of superfluid ^3He and its pressure dependence (Table 1 and Figure 2) exclude the contributions from $V'(N)$ (sum of interactions such as spin-spin interactions, electron-phonon interaction induced by electric polarization of the lattice, *etc.*) and possibly for this reason our estimates are about two times higher than experimental values. In view of these facts our estimates not only establish that the "mechanical strain" forced by zero-point force is the basic cause of bound pair formation in a SIF but also indicates that $V'(N)$ perturbations could be responsible for the supression of T_c below our estimates. We hope that this would be supported by studies related to the impact of these perturbations on T_c and its pressure dependence. Heiselberg *et. al.* [34] have summarized the important inferences of studies related to the impact of induced interactions such as BCS type attraction on the T_c deduced from bare fermion-fermion interaction. They identify that such interactions in liquid ^3He are responsible for the ABM state to be energetically more favorable than BM state, while in neutron matter they suppress the superfluid gap significantly. The effect has been studied in dilute spin 1/2 Fermi gas by Gorkov and Melik-Barkhurdarov [35] who find that T_c obtained from bare inter-particle interactions gets suppressed by a factor $(4e)^{1/3} \approx 2.2$. Evidently, all such effects of $V'(N)$ interactions (not included in deriving Eqn. 2) can be trusted for reducing the difference of our values of T_c with experiments (Table 1 and Figure 2). We plan to examine these effects in our future course of studies. Interestingly, similar results of pressure dependent T_c reported in

[19-22] are also about two times higher than experimental values but it appears that these studies leave no factor(s) which could help in getting better agreement with experiments.

Finally, it may be mentioned that we have limited information about the thermal expansion of superconductors [36] around T_c while the importance of its detailed and accurate measurements has been emphasized [37] soon after the discovery of high T_c superconductors. The observation of negative lattice expansion, anisotropic thermal expansion, change in hardness, *etc.* around T_c in a number of superconducting systems [28-33, and 38-40] not only re-emphasizes the importance of such studies but also indicates a relation of this effect with the onset of superfluidity in fermionic systems which naturally corroborates its mechanism as concluded by our theory [10].

4. Conclusion

The paper uses a relation obtained from our recently reported theoretical model [10] to estimate superfluid T_c of ^3He and its pressure dependence. The close agreement between our estimates and experimental results indicates the accuracy of our model and the microscopic mechanism of superfluidity in a SIF like liquid ^3He and electron fluid in widely different superconductors. As suggested in [28], we also believe that accurate measurements of different aspects related to modifications in lattice structure, *viz.* thermal expansion, changes in lattice parameters, hardening, change in sound velocity, *etc.* around superconducting T_c of widely different superconductors would be of great help in establishing the role of mechanical strain in the lattice as a basic component of the microscopic mechanism of superfluidity of different SIF and we hope that these would support our theory [10]. In this context it may be noted that liquid ^3He and liquid ^4He which do not have various complexities of electron fluid in conductors exhibit $-ve$ thermal expansion around superfluid T_c as predicted by their respective microscopic theories [10] and [11] based on our macro-orbital approach. In addition it is significant that our approach has no space for subjective considerations which provide widely different estimates of T_c as one may see with different T_c values (0 to 100 mK) [15-17, 19-22] estimated by using only one model (*the BCS picture*) for superfluid transition in liquid ^3He.

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